Abstract—In this paper we propose a digital timing recovery framework for mobile communication systems. We focus on iterative timing offset estimation based on a posteriori information in modern digital receivers. The proposed synchronizer maximizes on a turbo receiver’s ability to optimize its performance index in low Signal-to-Noise Ratio. The proposed scheme is made adaptive to any modulation as well as insensitive to carrier offsets apparent in receivers. It is shown through simulations that a derived sequential unconstrained minimization technique approaches a theoretical Cramer-Rao bound. It has also been shown that the proposed method outperforms conventional timing extraction methods with respect to jitter performance and synchronization convergence speed. This estimation method is suited to cellular communication receivers, and other related service applications where delay variation is a critical issue.

Index Terms—Iterative recovery, matched filter output, turbo complex, soft information.

I. INTRODUCTION

Timing recovery in low Signal-to-Noise Ratio (SNR) environments is one of the most critical receiver functions in digital communication systems. The output of the receiver filter \( r(t) \) must be sampled periodically at the sample rate and at the precise sampling instants \( t_m = mT + \tau_0 \)

where \( T \) is the symbol interval and \( \tau_0 \) is a nominal time delay that accounts for the propagation time of the signal from the transmitter to the receiver [1][2]. This requires an extraction of a clock signal and choosing on optimal timing phase within the received symbol interval for which the output of the receiver filter is maximum. In some wired systems such as computer networks, transmitters and receivers have been made to run off the same clock, but this implementation is not suited to wireless communication systems. The transmission of the clock frequency with the data signal is not the best solution due to bandwidth requirement constraints [3]. Timing recovery revolves around two main issues; Estimating timing information from received data symbols and implementing an optimal convergent algorithm which determines the steady-state location of the timing instants. Most classical timing estimations are derived from the maximum likelihood (ML) criterion, but have large computational complexity and usually work in data-aided (DA), decision-directed (DD) or non data-aided (NDA) synchronization principles [4][5]. In such situations, the timing recovery process can be separated from the decoding process with little penalty; timing recovery can use an instantaneous decision device to provide tentative decisions that are adequately reliable, which can then be used to estimate the timing error. In essence, the timing recovery process assumes that the neighboring symbols are independent at high SNR [2]. However, since turbo systems operate at low SNR values, classical estimators fail to provide good estimates of the synchronization parameters. In addition, soft information on bits or symbols is available at the decoder output. In the past decade, an iteratively decodable error-control code of unprecedented power with large coding gains has enable communication at very low SNR. Turbo principles enable timing recovery to be performed in lower SNR environment than ever before.

This paper is organized as follows. In section II, related work is presented. In section III, the problem statement is formulated and in section IV timing recovery framework is derived. Simulations are performed and conclusions drawn in sections V and VI respectively.

II. RELATED WORK

The idea of using soft information to estimate timing error parameters has already been applied in a number of contributions. In [6], iterative soft-decision directed timing estimation for turbo receivers was proposed with a focus on bit-interleaved coded modulation (BICM) scheme. In [7] turbo equalization has been used for iteratively approximating the joint-ML equalizer and decoder. In [8], joint timing recovery and ML equalizer and decoder have been presented. Though direct implementation of joint equalizer, decoder and timing recovery has increased the complexity of the receiver functions, incorporating an iterative procedure has maximized the joint likelihood function. Turbo estimation has been shown by [9] to be a special case of the expectation-maximization (EM) algorithm [10]. The work in [9] reveals a general theoretical framework for turbo synchronization that allows for parameter estimation procedures for carrier phase and frequency offset, as well as for timing offset and signal.

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amplitude. This contribution may work well in specific case where symbol synchronization based on assumption that the algorithm is insensitive to carrier offsets [11], is desired. In the presence of mobile communications, timing synchronization is more challenging. The derived timing estimate control signals must follow rapid changes in locating the maximum eye opening level. Thus, a rapidly converging algorithm for an adaptive synchronizer is an important requirement. The novel work in [13] showed a class of fast-converging timing recovery algorithms. However, this algorithm is sensitive to carrier frequency offset recovery. In this paper, an improved turbo timing recovery method for digital synchronization (which builds on the methods addressed in [6]-[8]) is proposed. We also examine work presented in [13] for binary baseband at low SNR for mobile communication systems.

III. PROBLEM FORMULATION

A. System modeling description

![System Block Model](image)

The considered transmitter is made up of turbo encoder, symbol mapper, and waveform shaping filter connected to the receiver through additive ambient white noise. The block diagram of such a system is shown in fig.1. The input binary information bit stream \( \{a_k\} \) are encoded using convolutional encoder then interleaved to reduce the effects of error bursts. The interleaver spreads bits over longer periods, making the transmitted bits look more “random.” The bit stream \( \{b_k\} \) is passed to a symbol mapper, yielding complex data symbols \( \{s_k\} \), which take values from a finite alphabet set \( A = \{a_1, a_2, \ldots, a_A\} \) (such as M-PSK,M-QAM,etc). Symbols are then passed through a unit energy square root raised cosine pulse with roll-off factor \( 0 < \alpha \leq 1 \) and output \( u(t) \). The raised cosine pulse shaping used is a compromise to an ideal time domain sinc function with limited bandwidth and low intersymbol interference (ISI) when sampled at correct time intervals. Assuming that \( u(t) \) is sent over complex envelope additive white Gaussian noise (AWGN) channel, \( n(t) \). For simplicity, we will assume that \( \{n(t)\} \) is a sequence of known power spectral density \( 2N_0 \) with independent and identically distributed (i.i.d) zero-mean complex random variables whose distribution is:

\[
n_t \rightarrow N_0(0, \sigma^2)
\]

Hence putting all these facts together, the baseband received signal, \( v(t) \) can be written as:

\[
v(t) = \sum_{k=0}^{A} a_k u(t - kT - \tau(t)) + n(t),
\]

(2)

where \( T \) and \( \tau(t) \) is the symbol period and the time varying delay respectively.

At the receiver, the output of a matched filter has a function, \( u(-t) \) and performs estimation of the transmitted signal. The matched filter output can be expressed as a convolution:

\[
r(t) = v(t) \otimes u(-t) = \sum_{k=-\infty}^{\infty} a_k h(t - kT + \tau(t)) + n(t),
\]

(3)

where \( h(t) \) is a raised cosine waveform with roll off, \( \alpha \) and is defined as:

\[
h(t) = u(t) \otimes u^*(t) = \frac{\sin(\pi t / T)}{\pi t / T} \cos(\pi \alpha t / T) - 1 - 4\alpha^2 t^2 / T^2.
\]

(4)

The matched filter output is sampled with a free running clock at instant \( t_m = mT_s \) with sampling interval \( T_s \leq T/(1 + \alpha) \), chosen to sufficiently avoid destructive aliasing effects, leading to samples:

\[
r_m = r(mT_s) = \sum_{k=-\infty}^{\infty} a_k \sum_{m} u(mT_s - kT - \tau_m) h^*(mT_s - s) - \sum_{m} n(mT_s) h^*(mT_s - s)
\]

(6)

We can rewrite equation (6) as;

\[
y_m = y(mT_s), w_m = w(mT_s), m = 0,1,2,\ldots, \text{ denotes the discrete-time index, } T_s \text{ is the sampling period, and } \tau_m = \tau(mT_s).
\]

Where:

\[
y_m = \sum_{k} a_k h(mT_s - kT - \tau_m) + w_m.
\]

(7)

In this paper, the problem addressed is the estimation of the unknown parameter, \( \tau \) based on soft-information estimates from a Soft-In, Soft-Out (SISO) turbo complex. In turbo decoding structure, serially concatenated codes provide the principal model for iterative decoding algorithm. The received symbol sequence goes through SISO modules repeatedly. The SISO modules’ processes involve detection, de-interleaving and decoding phases. Symbols at the output and input of the SISO modules are formulated as log-likelihood ratios. These ratios are namely, intrinsic, extrinsic, and a posteriori information of the transmitted bit symbols \( a_k \). In our interest we derive our timing recovery using a posteriori information at the receiver. Considering a nuisance vector \( \bf{a} = (a_1, a_2, \ldots, a_A) \), we aim at finding the trial value \( \hat{\tau} \) of the parameter vector, \( \tau \) which maximizes \( L(\hat{\tau}) \), such that

\[
L(\hat{\tau}) = \max L(\tau), \text{ and } \hat{\tau} = \arg \max_{\tau} L(\tau),
\]

(8)

where \( L \) is the likelihood non-decreasing function of the conditional probability and \( \hat{\tau} \) is the actual estimate timing offset. From theory of Information, turbo system’s entropy of SISO information is expressed as in (8) making our model appropriate. Neglecting the irrelevant terms
independent of the transmitted nuisance vector, \( a = \{a_1, a_2, \ldots, a_d\} \), and variable, \( r \) to be estimated, the log likelihood ratios (LLR) of (7) is denoted as;

\[
L(\hat{r}) = \ln p(r | a, \hat{\tau})
\]

(9)

Assuming that the vector, \( a = \{a_1, a_2, \ldots, a_d\} \) of the transmitted symbols, \( a_d \) has probability mass function denoted as \( p(a) \), and \( r = \{r_1, r_2, \ldots, r_R\} \), denotes the received signal vector dependent on \( a \). Applying Bayes rules to (8) yields an expanded form:

\[
L(\hat{r}) = \ln \left( \sum_a p(r | a, \hat{\tau}) p(a) \right).
\]

(10)

where \( p(r | a, \tau) = \prod_w p(r_w | a, \tau) \)

\[
= \prod_w \frac{1}{\sqrt{2\pi N_0 / T_s}} \exp \left( \frac{-(r_w - mT_s - n_m)^2}{2N_0 / T_s} \right),
\]

(11)

where

\[ n_m \approx n(mT_s) \]

Taylor series expansion gives the approximation of \( \exp(q) \approx 1 + q \) and \( \ln(1+q) \approx q \) and thus putting (11) into (10) and substituting (7) in the resulting expression yields an approximation [6]:

\[
\hat{L}(\hat{\tau}) =
\]

\[
= \sum_\tau \left\{ 2\Re \left[ \sum_{a \in A} p(a_k | a_k \gamma(kT + \tau)) \right] \right. \\
- \sum_k \left[ \sum_{a, a_k \in A} p(a_k, a_k^* a_k^* \gamma(kT - kT)) \right]
\]

(12)

We assume that the series approximations are valid in low signal to noise ratio in which turbo receiver operate. The evaluation of (12) requires knowledge of a priori information at the receiver; unfortunately, the a priori information is not directly available. The stochastic nature of most parameters in the received signal leads to definitions;

\[
\eta_i(r, \hat{\tau}) \approx \sum_{a_k \in A} a p(a_k = a | r, \hat{\tau})
\]

(13)

\[
\rho_k \approx \sum_{a_k \in A} |a|^2 p(a_k = a | r, \hat{\tau}),
\]

(14)

as the a posteriori mean and a posteriori mean square value of the transmitted symbol, \( a_k \) respectively. Here \( p(a_k = a | r, \hat{\tau}) \), denotes the marginal a posteriori probability (APP) of the kth channel symbol \( a_k \) conditioned on the observation of vector \( r = \{r_1, r_2, \ldots, r_R\} \) of the estimate \( \hat{\tau} \) at \( (k-1) \) th step, and \( a \) the m-possible values taken in the constellation, \( A \). Equation (13) depicts that for known marginal probabilities, a posteriori mean values would assume the size of all transmitted symbols. Without loss of generality, the proposed joint (turbo decoder and Equalizer) system models the received signal \( r \) in timing error as a Markov process. This enables the computation of marginal probabilities using the well known Bah-Jelinek-Cocke-Raviv (BSCR) algorithm. This novel algorithm has been applied in several literatures [7][8] and [14] for turbo equalization and control of symbol error rate, however its complexity leads to natural choice of Viterbi Algorithm [1]. In this work Viterbi Algorithm is used in soft symbol estimations.Viterbi Algorithm has the advantage solely, of its optimal trellis search speed. The maximally computed soft decision outputs are used as timing estimate system’s inputs.

B. A posteriori probability computation

Turbo complex receivers compute a posteriori probabilities on bits rather than on symbols. We make a valid assumption that coded bits are independent in every symbol, so that a priori probabilities on symbols may be approximated as:

\[
p(a_k) \approx \prod_{q=1}^{Q} p(c_k^q)
\]

(15)

and the corresponding posteriori probability as computed by SISO decoder given ith turbo iteration is denoted by:

\[
p(a_k | r, \hat{\tau}^{(l-1)}) \approx \prod_{q=1}^{Q} p(c_k^q | r, \hat{\tau}^{(l-1)})
\]

(16)

Where \( Q \) is the number of bits is contained in a symbol, \( a_k \), \( c_k^q \) is the qth bit of \( a_k \) and \( p(c_k^q | r, \hat{\tau}^{(l-1)}) \) is the bit a posteriori probabilities delivered by SISO decoder. Iterations performed on (16) compute both expectation-maximization symbol synchronization and turbo demodulations. In the next section we derive timing extraction algorithm in soft information outs depicted in (16).

IV. TIMING RECOVERY ALGORITHM

In this section the estimation of \( \hat{\tau} \) that maximizes equation (12) is developed. Fortunately in [12], several second order gradient solutions applied in optimal maximization problems have been proposed. Tabak and Kuo [12] reported the original work of Fiacco and McCormick in the field of sequential unconstrained minimization techniques (SUMT). It can be shown that modified Newton-raphson (MNR) yields suboptimum numerical solution to (12) in the shortest time possible. In application to rapidly changing receiver systems, this method would be a natural choice. We thus adopt it in our maximization problem.

Thus, substituting (13) and (14) in (12) and computing gradient solution:

\[
\left\{ \frac{\partial \hat{L}(\hat{\tau})}{\partial \hat{\tau}} \right\}_{\hat{\tau} = \hat{\tau}_k} = 0, \quad \text{or} \quad \sum_k \Re \left[ \gamma_k \hat{\gamma}(kT + \hat{\tau}) \right] = 0,
\]

(17)

where \( \hat{\gamma}(kT + \hat{\tau}) \) is the first order derivative of discrete matched filter output evaluated in \( kT + \hat{\tau} \). Incidentally, the function to be maximized has a quadratic behavior with \( \hat{\tau} \) close to \( \tau \). Solution to equation (17) requires recursive process. We notice also that higher order derivatives can
easily be ignored without loss of accuracy. Re-writing (17) as:

$$\hat{\tau}_{MAX}^{i} = \hat{\tau}^{i-1} - \left( \frac{\partial L(\hat{\tau})}{\partial \hat{\tau}} \right)_{\hat{\tau} = \hat{\tau}^{i-1}} \times \left( \frac{\partial^2 L(\hat{\tau})}{\partial \hat{\tau}^2} \right)^{-1} \hat{\tau}^{i-1}$$  

(18)

The first order derivative of the second term may be approximated as:

$$\frac{L(\hat{\tau}^{i-1} + \delta) - L(\hat{\tau}^{i-1} - \delta)}{2\delta}$$

$$= \frac{1}{\delta} \sum_{k} \Re \left\{ y \left( kT + \hat{\tau}^{i-1} + \delta \right) \right\}$$

(19)

while the second order derivative may take the form:

$$\frac{L(\hat{\tau}^{i-1} + \delta) - 2L(\hat{\tau}^{i-1}) + L(\hat{\tau}^{i-1} - \delta)}{\delta^2}$$

$$= \frac{1}{\delta^2} \sum_{k} \Re \left\{ 2y \left( kT + \hat{\tau}^{i-1} + \delta \right) \right\}$$

(20)

This is the basis of early-late gate synchronization principle performed on the output of sampled matched filter in fig.3. These samples are slightly earlier or later than previous estimate $\hat{\tau}^{i-1}$ by a predetermined step, $\delta$. On average, the output of the early-late samples will be smaller than the peak values. Optimal synchronization is reached when, the difference in side samples attain equilibrium. From equations (16) and (18), it can be seen that turbo iterations and symbol synchronization iterations are embedded as one process. At each iteration, the new timing estimate will be based on the discrete matched filter output and the posteriori probabilities from the previous iteration. The timing estimate convergence criterion depends on number of iterations in the symbol estimation systems. Further analysis of (18) shows that when more iterations are taken the expectation of random argument $\tau$ will yield optimal sampling time; that is turbo demodulator will estimate symbols from maximum eye opening samples of the matched filter output.

V. SIMULATION RESULTS

In this section the timing synchronization performance is investigated through simulations. At the transmitter, 2000 information bits is independently encoded using a rate-1/2 convolutional code. The bits are then interleaved using space-time interleavers. Modulation scheme used is 16-QAM. 500 symbols are generated from bit-symbol mapper. Square root raised cosine signaling with roll-off 0.2 is chosen and similar matched raised cosine filter is used at the receiver whose output is interpolated at taps set to 20. The discrete matched filter waveshape is used to derive timing estimates along with soft information from turbo systems. 18 turbo iterations are performed to attain timing synchronization level. Fig.4 gives the mean and variance of timing estimator so presented. Clearly, the higher the iterations, the closer to the Cramer-Rao bound and the more unbiased our timing estimator becomes. The mean values are increasingly linear at low iterations indicating good performance up to estimates closer to 0.35. Fig.5, presents the overall BER performance at low SNR, which is a normal operating condition with turbo receivers. It is evident that the proposed synchronizer and turbo detection operate as complimentary peers. While the synchronizer requires a low BER inorder to deliver unbiased and low variance timing estimates, the turbo demodulator uses accurate timing estimates in order to decrease the BER. Thus, at high turbo iterations, with increased SNR, the BER for the proposed timing estimator falls rapidly. In fig.6,a comparison in synchronization steady state is made with the well known Mueller and Muller’s Algorithm. Frames of 63-bit length were transmitted over low SNR additive Gaussian noise channel. At normalized offset of about 0.35, undesirable timing variations were noted at start-up time. The proposed algorithm however showed fewer variations during the data acquisition phase. This is desirable in mobile receiver systems.

VI. CONCLUSION

In this paper a soft information based timing extraction algorithm was presented. The idea of early-late gate synchronization was used. Turbo receiver system performance throughput was improved through iterative joint detection, decoding, equalization and synchronization.

Finally; it is noted that increasing iterations of soft information exchanges will not only converge BER but also generate good timing estimates at low SNR environments. Low jitter can be reported in this method showing its suitability for the design of cellular receivers.
Fig. 3: Discrete Interpolated matched filter output.

Fig. 4: Mean and Variance performance of proposed estimator.

VII. REFERENCES


