Kinematics Analysis and Workspace Investigation of a Proposed Parallel Mechanism for Multi-Planar Operations

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Abstract—This paper presents results for forward and reverse kinematics and workspace analysis of a proposed RPRPR 2.5D planar parallel manipulator using a geometric approach. The design has a potential to be adapted to diverse tasks implementable on a planar platform. The mechanism which is made up of two independent limbs of two bar linkages is capable of generating 2-DOF from the arm motion and an additional degree of freedom from an adjustable base which enhances its 2.5D capability. The workspace of the proposed mechanism has an overlapping range of $-\pi/2$ to $\pi/2$ for each rotating limb with a relatively small region of singularity. To demonstrate the coverage merit of the proposed design, a workspace comparison was carried out between the proposed design and that of prior research [1] which has a relatively complex but similar configuration. Despite its simple geometry, the proposed arm design has shown some promise in terms of workspace flexibility and optimization in comparison with other proposed planar configurations. The results were verified via simulation using Matlab and Python software packages.

Keywords - Parallel manipulator; Planar Mechanism; Kinematic; Workspace analysis; 2.5D domain

I. INTRODUCTION

In the past few years, the literature of parallel manipulator research has focused largely on optimal and economic designs of various classes of parallel mechanism ranging from planar through non-planar systems with an utmost goal to maximize workspace reachability amongst others. Simplicity of designs resulting in workspace enhancement coupled with versatility in terms of application domain of any given parallel mechanism has remained a major challenge in this body of knowledge.

One of the several ways of actualizing the gains of a 3D parallel mechanism and avoiding its relative design and analytical complexity with respect to a 2D mechanism is the development of a 2.5D integrated system. It enhances the dexterity of the arm mechanism and eliminates the likely effect of additional load constraint. It further improves the high payload to weight ratio and high structural rigidity amongst others.

A novel 3-DOF planar mechanism was proposed by Mir-Nasiri and G Hobashi [1] for PCB design and related applications. A planar 2-DOF parallel manipulator was proposed by Wu et al. [2] with actuation redundancy. Also Zhang and Zhang [3] proposed a novel 2-DOF parallel manipulator with three legs for a vehicle simulator while Dalla-Libera and Ishiguro [4] investigated the existence of non-singular assembly mode transitions in 2-DOF parallel manipulators.

An Optimization-based approach was proposed for force resolution of kinematically-redundant planar parallel manipulators by Boudreau and Nokleby [5]. Kucuk [6] proposed an optimization problem for the 3-DOF all revolute (RRR) fully planar parallel manipulator while Zi et al. [7] worked on cable parallel manipulators with and without a hybrid-driven planar five-bar mechanism. A robust nonlinear controller by Shang and Cong [8] was applied to a planar 2-DOF parallel manipulator with redundant actuation while Varalakshimi and Srinivas [9] presented an optimization methodology for achieving minimum actuation torques of a kinematically redundant planar parallel mechanism following a desired trajectory using binary coded genetic algorithms (GA).

A dexterity comparison was carried out by Kucuk [10] for seven degrees of freedom (3-DOF) Planar Parallel Manipulators with two kinematic chains (PPM2KCs) using genetic algorithms. While Li-xin and Yong-gang [11] investigated the effects of joint clearance on the dynamic performance of a planar 2-DOF pick-and-place parallel manipulator using multi-body system dynamics. Control of a redundantly actuated mechanism was presented by Muller and Hufnagel [12] while trail-tracking control approach of a 3-DOF parallel mechanism was proposed by Deqing [13]. A study on non-singular assembly mode transitions was presented by Coste [14] and Urizar et al. [15]. However, the latter provided a complete characterization of the cusp points in the 3-D joint space.
This paper presents preliminary results on the kinematics analysis and workspace investigation and comparison of a proposed planar parallel manipulator with earlier configurations. The 2-DOF arm design as shown in Figure 1 is made up of two independent limbs of two bar linkages. An adjustable platform housing a workpiece provides a 2.5D capability for the proposed mechanism. The domain of application of the proposed parallel system ranges from pantographing through laser cutting, general graphics application, inspection of work samples and a host of other planar related tasks.

![Proposed Planar Parallel Manipulator](image1)

**Fig. 1.** Proposed Planar Parallel Manipulator

### II. MODELLING

This section presents the modeling procedures for the kinematic analysis and work space investigation. The geometric modeling approach is deployed herein to achieve this. The kinematics study presented includes forward and inverse analysis.

#### A. Forward Kinematic Solution

Consider the sketch of two inter-locked links $a_1$ and $a_2$ set at a distance $d$ apart as shown in Figure 2.

![Sectional view of link mechanisms](image2)

**Fig.2.** Sectional view of link mechanisms

Assuming the angles $\theta_1$ and $\theta_2$ are orientations of links one and two respectively then the forward kinematics modeling procedure for the $x$ and $y$ axes is given as:

\[
x = a_1 \sin \theta_1 
\]

\[
y = a_1 \cos \theta_1 = a_2 \cos \theta_2 
\]

From (2)

\[
d = \frac{a_2 \cos \theta_2}{\cos \theta_1} 
\]

\[
d = a_1 \sin \theta_1 + a_2 \sin \theta_2 
\]

Where,

$d$ = the distance between the base of links 1 and 2.

Substituting (3) into (4) yields

\[
d = \frac{\cos \theta_1}{\cos \theta_1} a_2 \sin \theta_1 + a_2 \sin \theta_2
\]

Rearranging

\[
a_2 = \frac{d}{\cos \theta_1 \sin \theta_1 + \sin \theta_2 \cos \theta_1}
\]

Simplifying

\[
a_2 = \frac{d \cos \theta_1}{\cos \theta_2 \sin \theta_1 + \sin \theta_2 \cos \theta_1}
\]

Similarly

\[
a_1 = \frac{d \cos \theta_2}{\cos \theta_2 \sin \theta_1 + \sin \theta_2 \cos \theta_1}
\]

Substituting in (1)

\[
x = \frac{d \cos \theta_2 \sin \theta_1}{\cos \theta_2 \sin \theta_1 + \sin \theta_2 \cos \theta_1} 
\]

Similarly for $y$

\[
y = \frac{d \cos \theta_1 \cos \theta_2}{\cos \theta_2 \sin \theta_1 + \sin \theta_2 \cos \theta_1}
\]

Equations (6) and (7) are the respective forward kinematics solutions for the $x$ and $y$ axes.
B. Inverse Kinematic solution

The inverse solutions for the mechanism is as presented in (8) and (9).

\[ \theta_1 = \tan^{-1} \frac{x}{y} \]  
\[ \theta_2 = \tan^{-1} \frac{d-x}{y} \]  

(8)  
(9)

Where \(-\frac{\pi}{2} < \theta_1 < \frac{\pi}{2}\), \(-\frac{\pi}{2} < \theta_2 < \frac{\pi}{2}\) defines the range of analyticity for the intercepting links

C. Workspace Analysis

This subsection presents the workspace analysis of the proposed arm design. Figure 3 presents a view of the workspace coverage of the proposed 2-DOF arm.

Equations (10) and (11) are models representing the workspace coverage domain i.e. area \(A_1\) of the proposed planar design as seen in figure 3.

\[ A_1 = a_1 \theta_1 + a_2 (180 - \theta_2) - xy \]  

(10)

Where,

- \(a_1\)=length of link one and
- \(a_2\)=length of link two
- \(\theta_1\) = inclination of link one to the vertical axis
- \(\theta_2\) = inclination of link two to the vertical axis

Assuming

\[ R = \frac{y}{x} \]  

(11)

Where, \(R\)= ratio of lengths \(y\) and \(x\) i.e. end-effector's position relative to the \(x\) and \(y\) axes

\[ A_1 = 2x^2 (1 + R) \tan^{-1} R - x^2 R \]  

(12)

Equations (13) and (14) represents the model for a rectangular workspace profile.

\[ A_2 = 2xy \]  

(13)

(14)

To investigate the workspace coverage for both the proposed design and a rectangular profile i.e. \(A_1\) versus \(A_2\) assuming same link conditions, it implies from (11) and (12) that

\[ \frac{A_1}{A_2} = \frac{2x^2 (1 + \tan^{-1} R - x^2 R)}{2(1 + R^2) \tan^{-1} R - 0.5} \]  

(15)

Hence, to have a workspace with area \(A_1\) greater than area \(A_2\) assuming a Cartesian manipulator, it implies \(A_1/A_2\) must be greater than 1; Hence,

\[ \frac{(1 + R^2) \tan^{-1} R - 0.5}{R} > 1 \]  

(16)

III. RESULTS

This section presents a summary of the results relating the workspace of the proposed manipulator as presented herein with two other configurations: a rectangular configuration and a previously proposed 3-DOF planar design.
IV. DISCUSSION

Figure 4 depicts the relationship between the ratio \( \frac{A_1}{A_2} \) of the workspace coverage (for the proposed arm mechanism and the conventional Cartesian coordinate mechanism) versus \( R \) (the ratio of the end-effector relative position on the y and x axes from their respective origins). It could be seen that as the value of \( R \) increases, the ratio \( \frac{A_1}{A_2} \) also increases. The linear increment continues up until \( R=1 \) after which non-linear increment sets in disregarding the law of homogeneity. Figure 5 shows a comparative graph of both workspaces with same link lengths. Furthermore, it is worth noting that \( A_1 \) will be zero at \( a=x \) with \( \beta_1 = 0 \) and \( \beta_2 = \pi \).

Furthermore, on comparing the proposed design with a previously designed 3-DOF revolute joint planar manipulator presented in Figure 6, the new concept herein presents better workspace coverage. While Figure 7 represents the workspace
graphical outlook for the earlier design premised on 3-DOF, Figure 8 presents a comparative outlook where both workspaces are interplaced for comparison. The fully shaded outer plane which represents the workspace for the proposed design has a significant gain in space along both axes in comparison with the 3-DOF arm design whose workspace is inscribed and represented with dotted points. The proposed design though simple in terms of configuration, has a much enhanced workspace coverage and dexterity in application.

V. CONCLUSION

This paper has proposed a simple multi-application planar mechanism with an enhanced workspace domain in comparison with two prior configurations of the parallel sub-class. The dexterity of this mechanism is further enhanced with the attached adjustable base which facilitates its 2.5D domain. The workspace of the proposed mechanism has an overlapping range of -\( \pi/2 \) to \( \pi/2 \) for each rotating limb with a relatively small region of singularity. Both the forward and reverse kinematics of the proposed mechanism are less complex and more computationally efficient.

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REFERENCES


